BHARATI INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY
RESEARCH & DEVELOPMENT (BIJMRD)
(Open Access Peer-Reviewed International Journal)
DOI Link : https://doi.org/10.70798/Bijmrd/04080003

Available Online: www.bijmrd.com | BIJMRD Volume: 4 | Issue: 08 | August 2026 | e-ISSN: 2584-1890

Revisiting the Theory of Tripolar HyperFuzzy Graphs

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Abstract: *Graph-based models provide a fundamental mathematical framework for representing relational systems in terms of vertices and their connections. Fuzzy sets extend classical set theory by assigning each element a membership degree in the interval [0,1], thereby allowing partial membership rather than restricting membership to binary values.Through the use of HyperStructures [17], Fuzzy Sets have been generalized to HyperFuzzy Sets [18–20]. Furthermore, the HyperFuzzy Set framework has been extended to graph-theoretic settings, leading to the development of HyperFuzzy Graphs [39–41].Motivated by these developments, this paper investigates tripolar extensions of the hyperfuzzy framework. In particular, we study Tripolar HyperFuzzy Sets and Tripolar HyperFuzzy Graphs, which integrate the three-pole representation of tripolar fuzzy models with the set-valued membership structure underlying HyperFuzzy Sets and HyperFuzzy Graphs.*

Keywords: *HyperFuzzy Set, HyperFuzzy Graph, Tripolar Fuzzy Set.*

1. Introduction: Fuzzy set theory offers a mathematical framework for representing partial membership by assigning each element a degree in the interval [0,1], instead of restricting membership to the two crisp values used in classical set theory [1]. Since its introduction, fuzzy set theory has become an important tool for describing vagueness and uncertainty and has motivated the development of many generalized uncertainty models. Representative examples include Neutrosophic Sets [2–4], Plithogenic Sets [5,6], and several other extended frameworks.

A significant line of development concerns the representation of information from multiple poles or viewpoints. Bipolar Fuzzy Sets describe uncertainty through two contrasting perspectives, typically expressed by positive and negative membership components [7–9]. This idea has subsequently been extended to related models, including Bipolar Neutrosophic Sets [9,10], m-Polar Fuzzy Sets [11,12], and other multipolar structures. Within this broader setting, a Tripolar Fuzzy Set may be interpreted as a three-pole realization of an m-polar framework [13–16]. The introduction of three distinct components allows information to be represented from a richer collection of perspectives than is possible in the bipolar case. Nevertheless, the literature contains several formulations of Tripolar Fuzzy Sets, reflecting different interpretations and mathematical treatments of the three poles.

Another important direction is obtained by incorporating higher-order structural ideas. Through the use of HyperStructures [17], Fuzzy Sets have been generalized to HyperFuzzy Sets [18-20], while Neutrosophic Sets have similarly been extended to HyperNeutrosophic Sets [21,22]. These constructions can be developed further within the framework of SuperHyperStructures [23], leading to concepts such as SuperHyperFuzzy Sets and SuperHyperNeutrosophic Sets [24,25]. Such generalized structures are potentially useful for representing layered, interconnected, and highly uncertain information, and they have therefore received increasing attention in recent research.

Graph-based uncertainty models have developed in parallel with these set-theoretic extensions. A Fuzzy Graph generalizes a classical graph by assigning membership degrees to vertices and edges, thereby allowing uncertain, vague, or only partially specified relationships to be represented mathematically [26-28]. This framework has inspired several further extensions, including Intuitionistic Fuzzy Graphs [29], Neutrosophic Graphs [30-32], Interval-Valued Neutrosophic Graphs [33-35], and Plithogenic Graphs [36-38]. In addition, the HyperFuzzy Set framework has been incorporated into graph theory, giving rise to the notion of a HyperFuzzy Graph [39-41].

Motivated by these developments, this paper investigates tripolar extensions of the hyperfuzzy framework. In particular, we study Tripolar HyperFuzzy Sets and Tripolar HyperFuzzy Graphs, which combine the three-pole representation of tripolar fuzzy models with the set-valued membership structure underlying HyperFuzzy Sets and HyperFuzzy Graphs. These concepts are intended to provide a broader framework for describing relational systems involving both multiperspective uncertainty and hyperfuzzy information.

2. Preliminaries: This section introduces the fundamental concepts and notation required for the subsequent discussion. Unless otherwise specified, every graph considered throughout this paper is assumed to be finite.

2.1. Tripolar Fuzzy Set and Graph: Tripolar fuzzy models describe uncertain information by means of three coordinated components associated with positive, neutral, and counter-property perspectives [13-16]. Unlike an ordinary fuzzy description based on a single membership grade, this representation permits three distinct viewpoints to be recorded simultaneously. When the same principle is transferred to graphs, tripolar values are assigned to both vertices and edges, and appropriate incidence conditions are imposed so that edge information remains compatible with its endpoints.

Definition 1 (Tripolar Fuzzy Set) (cf. [13-16]). Let X be a nonempty set. For the Rao-type formulation adopted in this paper, a Tripolar Fuzzy Set (TFS) A on X is represented by the structure

$$A = \{(x, \rho_A(x), \nu_A(x), \delta_A(x)) : x \in X\},$$

whose three component functions are given by

$$\rho_A, \nu_A : X \to [0,1], \qquad \delta_A : X \to [-1,0],$$

subject to the following admissibility condition for every $x \in X$.

$$0 \leq \rho_A(x) + \nu_A(x) \leq 1.$$

Here, $\rho_A(x)$ gives the positive membership degree, $\nu_A(x)$ describes the neutral, indeterminate, or irrelevant degree, while $\delta_A(x)$ expresses the degree associated with the corresponding counter-property. Thus, each element $x \in X$ is represented by the tripolar value

$$A(x) = (\rho_A(x), \nu_A(x), \delta_A(x)).$$

For two admissible tripolar values

$$a = (a_p, a_0, a_c), \qquad b = (b_p, b_0, b_c),$$

the componentwise order used throughout this paper is defined by

$$a \preccurlyeq_T b \quad \Leftrightarrow \quad a_p \leq b_p, \qquad a_0 \geq b_0, \qquad a_c \geq b_c.$$

With respect to this order, the corresponding meet is

$$a \wedge_T b = \left(\min\{a_p, b_p\}, \max\{a_0, b_0\}, \max\{a_c, b_c\}\right).$$

Definition 2 (Fuzzy Graph). Let G* = (V,E) be a finite simple graph. A Fuzzy Graph on the underlying graph G* is a pair $G_F = (\sigma, \mu)$, consisting of the mappings

σ : V → [0,1],

μ : E → [0,1],

such that, for each edge {u,v} ∈ E,

μ({u,v}) ≤ min{σ(u), σ(v)}.

The value σ(v) expresses the membership degree of a vertex v, whereas μ({u,v}) gives the membership grade associated with the edge {u,v}. The inequality above guarantees incidence compatibility: the membership assigned to an edge cannot exceed the membership degree of either incident vertex.

Definition 3 (Tripolar Fuzzy Graph). Let

$$G = (V, E), \qquad E \subseteq \binom{V}{2},$$

be a finite simple graph. Assume that

$$A_V = (\rho_V, \nu_V, \delta_V)$$

is a Tripolar Fuzzy Set on V, and that

$$A_E = (\rho_E, \nu_E, \delta_E)$$

is a Tripolar Fuzzy Set on E. Write

$$\rho_V, \nu_V : V \to [0,1], \qquad \delta_V : V \to [-1,0],$$

$$\rho_E, \nu_E : E \to [0,1], \qquad \delta_E : E \to [-1,0],$$

where

$$0 \le \rho_V(v) + \nu_V(v) \le 1 \quad (v \in V),$$

and

$$0 \le \rho_E(e) + \nu_E(e) \le 1 \quad (e \in E).$$

The structure

$$\mathcal{G} = (G, A_V, A_E)$$

is called a Tripolar Fuzzy Graph (TFG) provided that, for every edge $e = \{u, v\} \in E$, the following incidence-compatibility condition is satisfied:

$$A_E(e) \preccurlyeq_T A_V(u) \wedge_T A_V(v).$$

Equivalently, in componentwise form, for every $\{u, v\} \in E$, the condition can be written as

$$\rho_E(\{u,v\}) \le \min\{\rho_V(u), \rho_V(v)\},$$

$$\nu_E(\{u,v\}) \ge \max\{\nu_V(u), \nu_V(v)\},$$

together with

$$\delta_E(\{u,v\}) \ge \max\{\delta_V(u), \delta_V(v)\}.$$

The positive component obeys the usual incidence restriction from fuzzy graph theory. The neutral and counter-property components, however, follow the reversed directions prescribed by the tripolar order above, so their inequalities appear with the opposite orientation. Taken together, these three componentwise conditions ensure that the tripolar information carried by an edge is compatible with the tripolar information assigned to both incident vertices.

2.2. HyperFuzzy Set and HyperFuzzy Graph: HyperFuzzy Sets replace a single fuzzy membership degree by a nonempty set of admissible membership degrees. HyperFuzzy Graphs transfer this set-valued membership mechanism to vertices and edges [49,50].

Definition 4 (HyperFuzzy Set) [39,40]. Let $X \neq \varnothing$ and let *I=[0,1]*. We define

$$\tilde{\mathcal{P}}(I) = \{A \subseteq I : A \neq \varnothing\}.$$

A *HyperFuzzy Set*$\tilde{A}$ on X is a mapping

$$\tilde{\mu}_A : X \to \tilde{\mathcal{P}}([0,1]).$$

Thus, each $x \in X$ is assigned a nonempty set $\tilde{\mu}_A(x) \subseteq [0,1]$ of possible membership degrees.

For every nonempty bounded real set A, let $U(A)$ denote its supremum and let $L(A)$ denote its infimum.

Definition 5 (HyperFuzzy Graph) [39,40]. Let $G_0 = (V, E)$ be a finite simple graph. A *HyperFuzzy Graph* on G_0 is a triple

$$\tilde{G} = (G_0, \tilde{\sigma}, \tilde{\mu}),$$

where

$$\tilde{\sigma} : V \to \tilde{\mathcal{P}}([0,1]),$$

$$\tilde{\mu} : E \to \tilde{\mathcal{P}}([0,1]).$$

and, for every edge $e = \{u, v\} \in E$,

$$U(\tilde{\mu}(e)) \le \min\{U(\tilde{\sigma}(u)), U(\tilde{\sigma}(v))\}.$$

Because all assigned sets are nonempty subsets of [0,1], the quantities appearing in this condition are well-defined.

3. Results: Tripolar HyperFuzzy Graph: We now combine the tripolar structure of Definition 1 with the set-valued membership mechanism of HyperFuzzy Sets. The upper endpoint functional U is used for the positive component, whereas the lower endpoint functional L is used for the neutral and counter-property components, consistently with the reversed order of the latter two coordinates in the tripolar order $\preccurlyeq_T$.

Definition 6 (Tripolar HyperFuzzy Set). Let X be a nonempty set, and define

$$\mathcal{H}_p = \{A \subseteq [0,1]: A \neq \varnothing\}, \qquad \mathcal{H}_c = \{A \subseteq [-1,0]: A \neq \varnothing\}.$$

A *Tripolar HyperFuzzy Set* (THFS) $\tilde{A}$ on X is a triple

$$\tilde{A} = (\tilde{\rho}_A, \tilde{\nu}_A, \tilde{\delta}_A),$$

where

$$\tilde{\rho}_A, \tilde{\nu}_A: X \to \mathcal{H}_p, \qquad \tilde{\delta}_A: X \to \mathcal{H}_c,$$

and, for every $x \in X$,

$$U(\tilde{\rho}_A(x)) + U(\tilde{\nu}_A(x)) \leq 1.$$

Thus, each $x \in X$ is represented by the tripolar set-valued membership profile

$$\tilde{A}(x) = \left(\tilde{\rho}_A(x), \tilde{\nu}_A(x), \tilde{\delta}_A(x)\right).$$

The three components describe positive, neutral (or indeterminate/irrelevant), and counter-property hyperfuzzy information, respectively. If every assigned set is a singleton, then this definition reduces exactly to the Tripolar Fuzzy Set of Definition 1.

Definition 7 (Tripolar HyperFuzzy Graph). Define

$$\mathcal{H}_p = \{A \subseteq [0,1]: A \neq \varnothing\}.$$

$$\mathcal{H}_c = \{A \subseteq [-1,0]: A \neq \varnothing\}.$$

Let $G_0 = (V, E)$ be a finite simple graph. Consider the vertex maps

$$\tilde{\rho}_V, \tilde{\nu}_V: V \to \mathcal{H}_p,$$

$$\tilde{\delta}_V: V \to \mathcal{H}_c.$$

and the edge maps

$$\tilde{\rho}_E, \tilde{\nu}_E: E \to \mathcal{H}_p,$$

$$\tilde{\delta}_E: E \to \mathcal{H}_c.$$

Assume that

$$U(\tilde{\rho}_V(v)) + U(\tilde{\nu}_V(v)) \leq 1, v \in V,$$

and

$$U(\tilde{\rho}_E(e)) + U(\tilde{\nu}_E(e)) \leq 1, e \in E.$$

Write

$$\tilde{A}_V = (\tilde{\rho}_V, \tilde{\nu}_V, \tilde{\delta}_V).$$

$$\tilde{A}_E = (\tilde{\rho}_E, \tilde{\nu}_E, \tilde{\delta}_E).$$

The structure

$$\tilde{G} = (G_0, \tilde{A}_V, \tilde{A}_E)$$

is called a *Tripolar HyperFuzzy Graph* (THFG) if, for every $e = \{u, v\} \in E$,

$$U(\tilde{\rho}_E(e)) \leq \min\{U(\tilde{\rho}_V(u)), U(\tilde{\rho}_V(v))\},$$

$$L(\tilde{\nu}_E(e)) \geq \max\{L(\tilde{\nu}_V(u)), L(\tilde{\nu}_V(v))\},$$

and

$$L\left(\tilde{\delta}_E(e)\right) \geq \max\{L\left(\tilde{\delta}_V(u)\right), L\left(\tilde{\delta}_V(v)\right)\}.$$

If every assigned set is a singleton, Definition 7 reduces exactly to the Tripolar Fuzzy Graph of Definition 3.

Theorem 1 (Well-definedness of the Tripolar HyperFuzzy Graph). Let $G_0 = (V, E)$ be a finite simple graph, and suppose that the vertex maps $\tilde{\rho}_V, \tilde{\nu}_V, \tilde{\delta}_V$ in Definition 6 have nonempty values in their prescribed intervals and satisfy

$$U(\tilde{\rho}_V(v)) + U(\tilde{\nu}_V(v)) \leq 1 (v \in V).$$

For each edge $e = \{u, v\} \in E$, define

$$\tilde{\rho}_E(e) = \{\min(r_u, r_v) : r_u \in \tilde{\rho}_V(u),\ r_v \in \tilde{\rho}_V(v)\},$$

$$\tilde{\nu}_E(e) = \{\max(n_u, n_v) : n_u \in \tilde{\nu}_V(u),\ n_v \in \tilde{\nu}_V(v)\},$$

and

$$\tilde{\delta}_E(e) = \{\max(d_u, d_v) : d_u \in \tilde{\delta}_V(u),\ d_v \in \tilde{\delta}_V(v)\}.$$

Then these maps are well-defined and determine a Tripolar HyperFuzzy Graph $\tilde{G} = (G_0, \tilde{A}_V, \tilde{A}_E)$.

Proof. Fix $e = \{u, v\} \in E$. Since all vertex-value sets are nonempty, each of the three sets defining $\tilde{\rho}_E(e)$, $\tilde{\nu}_E(e)$, and $\tilde{\delta}_E(e)$ is nonempty. The functions min and max preserve the intervals $[0,1]$ and $[-1,0]$, respectively. Hence

$$\tilde{\rho}_E(e), \tilde{\nu}_E(e) \in \mathcal{H}_p.$$

$$\tilde{\delta}_E(e) \in \mathcal{H}_c.$$

Moreover, these definitions are symmetric in u and v. Therefore, they depend only on the unordered edge $\{u, v\}$ and not on an ordering of its endpoints.

For arbitrary nonempty bounded real sets A and B, one has

$$U(\{\min(a, b) : a \in A,\ b \in B\}) = \min\{U(A), U(B)\},$$

and

$$L(\{\max(a, b) : a \in A,\ b \in B\}) = \max\{L(A), L(B)\}.$$

Consequently,

$$U(\tilde{\rho}_E(e)) = \min\{U(\tilde{\rho}_V(u)), U(\tilde{\rho}_V(v))\},$$

$$L\big(\tilde{\nu}_E(e)\big) = \max\{L\big(\tilde{\nu}_V(u)\big), L\big(\tilde{\nu}_V(v)\big)\},$$

and

$$L\left(\tilde{\delta}_E(e)\right) = \max\{L\left(\tilde{\delta}_V(u)\right), L\left(\tilde{\delta}_V(v)\right)\}.$$

Thus all three edge-compatibility conditions in Definition 7 hold, in fact with equality.

It remains to verify the edge admissibility condition. For $x \in \{u, v\}$, put

$$p_x = U\big(\tilde{\rho}_V(x)\big).$$

$$n_x = U\big(\tilde{\nu}_V(x)\big).$$

The canonical construction gives

$$U\big(\tilde{\rho}_E(e)\big) = \min\{p_u, p_v\}.$$

$$U\big(\tilde{\nu}_E(e)\big) = \max\{n_u, n_v\}.$$

Choose $w \in \{u, v\}$ such that $n_w = \max\{n_u, n_v\}$. Then

$$U\big(\tilde{\rho}_E(e)\big) + U\big(\tilde{\nu}_E(e)\big) = \min\{p_u, p_v\} + n_w \le p_w + n_w \le 1.$$

Hence the edge tripolar hyperfuzzy value satisfies the required admissibility condition. Since e was arbitrary, the construction defines valid edge maps on all of E. Consequently, $\tilde{G}$ is a Tripolar HyperFuzzy Graph. This completes the proof.

4. Conclusion This paper investigated tripolar extensions of the hyperfuzzy framework. In particular, we studied Tripolar HyperFuzzy Sets and Tripolar HyperFuzzy Graphs, which integrate the three-pole representation of tripolar fuzzy models with the set-valued membership structure underlying HyperFuzzy Sets and HyperFuzzy Graphs. We hope that future studies will further extend the concepts introduced in this paper by incorporating structures such as HyperGraphs [44,45] and SuperHyperGraphs [46,47].

Funding: This study did not receive any financial or external support from organizations or individuals.

Acknowledgments: We extend our sincere gratitude to everyone who provided insights, review, inspiration, and assistance throughout this research. We particularly thank our readers for their interest and acknowledge the authors of the cited works for laying the foundation that made our study possible. We also appreciate the support from individuals and institutions that provided the resources and infrastructure needed to produce and share this paper. Finally, we are grateful to all those who supported us in various ways during this project.

Data Availability: This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval: As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Use of Generative AI and AI-Assisted Tools: I use generative AI and AI-assisted tools for tasks such as English grammar checking, and I do not employ them in any way that violates ethical standards.

Conflicts of Interest: The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer (Conversion from LaTeX to Word): Since this manuscript was generated by converting LaTeX source into a Word document, minor formatting discrepancies may have occurred. We have made every effort to eliminate errors as much as possible, but we kindly ask for your understanding in advance.

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Citation: **Fujita. T.,** (2026) “Revisiting the Theory of Tripolar HyperFuzzy Graphs”, *Bharati International Journal of Multidisciplinary Research & Development (BIJMRD)*, Vol-4, Issue-08, August-2026.