



Centrality of the Stern–Gerlach Experiment in Quantum Mechanics

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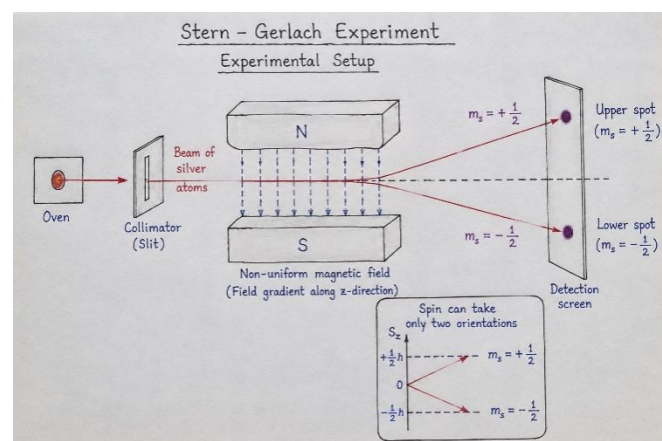
Abstract: *The Stern–Gerlach experiment holds key role for students, particularly those studying quantum mechanics, modern physics, atomic physics, and physics education. It is one of the simplest experiments that demonstrates that the microscopic world does not behave accordingly with dynamics of classical physics, thus the Stern–Gerlach experiment allows students to follow a logical progression: Classical expectation → Experimental observation → Failure of classical explanation → Quantum hypothesis → Spin states → Mathematical formulation → Quantum measurement.*

Keywords: *Quantum Mechanics, quantum mechanics, modern physics, atomic physics, physics education.*

Introduction: The experiment was conducted by Otto Stern and Walther Gerlach in 1922, is one of the landmark experiments in the evolution of quantum mechanics. It provided direct experimental evidence that microscopic physical quantities, particularly angular momentum, do not always possess arbitrary continuous values but can exhibit quantized values. The experiment also demonstrated the quantization of the magnetic moment associated with atomic angular momentum and laid an important foundation for the modern concept of quantum spin.

At the time of the experiment, the old quantum theory was struggling to explain several atomic phenomena. Classical physics predicted that the magnetic moments of atoms could have continuously distributed orientations in an external magnetic field. Stern and Gerlach designed an experiment capable of testing this prediction by observing the deflection of a beam of Silver atoms (Ag-47) passing through a non-uniform magnetic field.

Experimental Arrangement



In the original experiment, a beam of Ag-47 atoms was produced by heating silver in an oven. The atoms emerged through a narrow slit and formed a collimated atomic beam. This beam was then passed through a specially designed non-uniform magnetic field produced by shaped magnetic poles. Finally, the atoms reached a detector or photographic plate.

A classical model would predict a continuous spread of atoms on the detector because the

magnetic moments could supposedly have any orientation relative to the magnetic field. However, Stern and Gerlach observed something fundamentally different: the beam separated into two distinct components.

The simplified experimental sequence can be represented as:

Silver (Ag-47) atoms → Collimation → non-uniform magnetic field → Spatial separation → Detector

Classically, mathematics tells us:

$$F_z = \mu_z \frac{\partial B_z}{\partial z}.$$

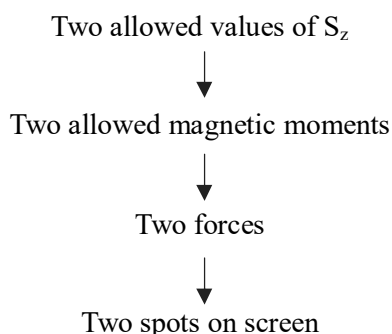
Whereas, Quantum mechanically, mathematics tells us:

$$S_z = m_s \hbar$$

and for spin- $\frac{1}{2}$,

$$m_s = \pm \frac{1}{2}.$$

Therefore,



Mathematical basis of the Experiment

Physical idea: The experiment demonstrates that the component of an atomic magnetic moment is *quantized*, along a chosen direction.

A beam of neutral atoms is made to pass through a non-uniform magnetic field. Classically, one would expect a continuous distribution of deflections because the magnetic moments could have arbitrary orientations.

Quantum mechanically, for a spin- $\frac{1}{2}$ particle,

$$S_z = \pm \frac{\hbar}{2},$$

so only two values of the magnetic moment component are possible. Consequently, the beam splits into two components. Spin- $\frac{1}{2}$ and Pauli matrices: For a spin- $\frac{1}{2}$ particle, the spin angular momentum operator is

$$\hat{S} = \frac{\hbar}{2} \sigma$$

Where,

$$\sigma = (\sigma_x, \sigma_y, \sigma_z)$$

are the Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

and

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Therefore,

$$\hat{S}_x = \frac{\hbar}{2}\sigma_x, \hat{S}_y = \frac{\hbar}{2}\sigma_y, \hat{S}_z = \frac{\hbar}{2}\sigma_z.$$

Eigenvalues of S_z : The measurement of spin along the z-axis is described by

$$\hat{S}_z\chi = \lambda\chi.$$

Since

$$\hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

we solve

$$\frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \lambda \begin{pmatrix} a \\ b \end{pmatrix}.$$

This gives

$$\lambda_+ = +\frac{\hbar}{2}$$

and

$$\lambda_- = -\frac{\hbar}{2}.$$

The corresponding normalized eigenstates are

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

and

$$\chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Thus,

$$S_z\chi_{\pm} = \pm \frac{\hbar}{2}\chi_{\pm}$$

This is the mathematical origin of the two possible outcomes in the Stern–Gerlach experiment.

Magnetic moment operator: For a spin- $\frac{1}{2}$ particle, the magnetic moment is related to spin by

$$\hat{\mu} = -g \frac{e}{2m} \hat{S}$$

for an electron.

Since

$$\hat{S} = \frac{\hbar}{2}\sigma,$$

we obtain

$$\hat{\mu} = -g \frac{e\hbar}{4m} \sigma.$$

Introducing the Bohr magneton,

$$\mu_B = \frac{e\hbar}{2m_e},$$

we can write

$$\hat{\mu} = -g\mu_B \frac{\sigma}{2}$$

for the electron.

For $g \simeq 2$,

$$\hat{\mu} \simeq -\mu_B \sigma.$$

Hence,

$$\hat{\mu}_z = -\mu_B \sigma_z.$$

Therefore,

$$\hat{\mu}_z \chi_+ = -\mu_B \chi_+$$

and

$$\hat{\mu}_z \chi_- = +\mu_B \chi_-.$$

So, the two allowed magnetic-moment projections are

$$\mu_z = \pm \mu_B$$

with the sign depending on the electron-moment convention.

Magnetic interaction energy: The interaction energy between a magnetic moment and a magnetic field is

$$U = -\mu \cdot B.$$

Therefore, the quantum-mechanical interaction Hamiltonian is

$$\hat{H}_{\text{int}} = -\hat{\mu} \cdot B.$$

Substituting the magnetic moment operator,

$$\hat{H}_{\text{int}} = g\mu_B \frac{\sigma}{2} \cdot B.$$

Thus,

$$\hat{H}_{\text{int}} = \frac{g\mu_B}{2} \sigma \cdot B.$$

This is the key Hamiltonian describing the interaction of spin with the magnetic field.

Magnetic field in the experimental setup: Let the magnetic field be predominantly in the z-direction:

$$B \simeq B_z(z) \hat{z}.$$

Then

$$\sigma \cdot B = \sigma_z B_z(z).$$

Hence,

$$\hat{H}_{\text{int}} = \frac{g\mu_B}{2} B_z(z) \sigma_z.$$

For $g = 2$,

$$\hat{H}_{\text{int}} = \mu_B B_z(z) \sigma_z.$$

Notice that the magnetic field depends on z . This spatial dependence is what produces a force.

Total Hamiltonian: The atom also has translational kinetic energy.

Therefore,

$$\hat{H} = \frac{\hat{p}^2}{2M} + \hat{H}_{\text{int}}$$

where M is the atomic mass.

Thus,

$$\hat{H} = \frac{\hat{p}^2}{2M} + \frac{g\mu_B}{2} B_z(z) \sigma_z.$$

The time-dependent Schrödinger equation is

$$i\hbar \frac{\partial}{\partial t} \Psi(r, t) = \left[-\frac{\hbar^2}{2M} \nabla^2 + \frac{g\mu_B}{2} B_z(z) \sigma_z \right] \Psi(r, t)$$

where Ψ is now a **two-component spinor**.

Spinor wavefunction: Write the wavefunction as

$$\Psi(r, t) = \begin{pmatrix} \psi_+(r, t) \\ \psi_-(r, t) \end{pmatrix}.$$

Since

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

the Schrödinger equation becomes

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \left[-\frac{\hbar^2}{2M} \nabla^2 + \frac{g\mu_B B_z}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}.$$

Multiplying the matrix gives two independent equations:

$$i\hbar \frac{\partial \psi_+}{\partial t} = \left[-\frac{\hbar^2}{2M} \nabla^2 + \frac{g\mu_B}{2} B_z(z) \right] \psi_+$$

and

$$i\hbar \frac{\partial \psi_-}{\partial t} = \left[-\frac{\hbar^2}{2M} \nabla^2 - \frac{g\mu_B}{2} B_z(z) \right] \psi_-.$$

These equations are fundamental to the Stern–Gerlach derivation.

Assume a linear magnetic-field gradient: Inside the Stern–Gerlach magnet, approximate the magnetic field by

$$B_z(z) = B_0 + B'z$$

where

$$B' = \frac{\partial B_z}{\partial z}.$$

Therefore,

$$\frac{g\mu_B}{2} B_z = \frac{g\mu_B}{2} (B_0 + B'z).$$

The two equations become

$$i\hbar \frac{\partial \psi_+}{\partial t} = \left[-\frac{\hbar^2}{2M} \nabla^2 + \frac{g\mu_B B_0}{2} + \frac{g\mu_B B'}{2} z \right] \psi_+$$

and

$$i\hbar \frac{\partial \psi_-}{\partial t} = \left[-\frac{\hbar^2}{2M} \nabla^2 - \frac{g\mu_B B_0}{2} - \frac{g\mu_B B'}{2} z \right] \psi_-.$$

The constant B_0 contributes only a phase to each spin component, whereas the terms proportional to z generate forces.

Force obtained from the Hamiltonian

The force operator is

$$\hat{F}_z = -\frac{\partial \hat{H}}{\partial z}.$$

Therefore,

$$\hat{F}_z = -\frac{g\mu_B}{2} \frac{\partial B_z}{\partial z} \sigma_z.$$

Hence,

$$\hat{F}_z = -\frac{g\mu_B}{2} B' \sigma_z.$$

For an eigenstate of σ_z ,

$$\sigma_z \chi_{\pm} = \pm \chi_{\pm}.$$

Consequently,

$$F_z^{(+)} = -\frac{g\mu_B}{2} B'$$

and

$$F_z^{(-)} = +\frac{g\mu_B}{2} B'.$$

For $g = 2$,

$$F_z^{(+)} = -\mu_B B'$$

and

$$F_z^{(-)} = +\mu_B B'.$$

The important point is that the two spin states experience **opposite forces**.

Classical-looking equations of motion emerging from quantum mechanics: For each spin component,

$$M \frac{d^2 z}{dt^2} = F_z.$$

Thus,

$$M \frac{d^2 z_+}{dt^2} = -\frac{g\mu_B}{2} B'$$

and

$$M \frac{d^2 z_-}{dt^2} = +\frac{g\mu_B}{2} B'.$$

Therefore,

$$a_+ = -\frac{g\mu_B}{2M} B'$$

and

$$a_- = +\frac{g\mu_B}{2M} B'.$$

For $g = 2$,

$$a_{\pm} = \mp \frac{\mu_B B'}{M}.$$

This demonstrates how the Schrödinger equation produces two spatially separated components.

Wave-packet description: Suppose the incoming atom is initially described by a spatial wave packet

$$\phi(r)$$

and its spin state is

$$\chi = a\chi_+ + b\chi_-.$$

The initial state is therefore

$$\Psi(r, 0) = \phi(r)(a\chi_+ + b\chi_-).$$

Explicitly,

$$\Psi(r, 0) = \begin{pmatrix} a\phi(r) \\ b\phi(r) \end{pmatrix}.$$

Normalization requires

$$|a|^2 + |b|^2 = 1.$$

Evolution inside the magnetic field: Because the two spin components experience different potentials,

$$V_{\pm}(z) = \pm \frac{g\mu_B}{2} B_z(z),$$

the initial wave packet evolves into two different wave packets:

$$\Psi(r, t) = a\psi_+(r, t)\chi_+ + b\psi_-(r, t)\chi_-.$$

The centers of these wave packets move according to

$$z_+(t) = z_0 + v_{z0}t + \frac{1}{2}a_+t^2$$

and

$$z_-(t) = z_0 + v_{z0}t + \frac{1}{2}a_-t^2.$$

If initially

$$z_0 = 0, v_{z0} = 0,$$

then

$$z_{\pm}(t) = \frac{1}{2}a_{\pm}t^2.$$

Hence,

$$z_{\pm}(t) = \mp \frac{g\mu_B B'}{4M} t^2.$$

For $g = 2$,

$$z_{\pm}(t) = \mp \frac{\mu_B B'}{2M} t^2.$$

Separation of the two beams: The distance between the centers is

$$\Delta z = |z_+ - z_-|.$$

Therefore,

$$\Delta z = \frac{g\mu_B |B'|}{2M} t^2.$$

For $g = 2$,

$$\Delta z = \frac{\mu_B |B'|}{M} t^2.$$

This is the separation while the particles are inside the magnetic field under the constant-gradient approximation.

Expressing the time in terms of magnet length

Let

- L = length of the magnet,
- v_x = velocity of the atoms in the x -direction.

Then

$$t = \frac{L}{v_x}.$$

Therefore,

$$\Delta z = \frac{g\mu_B |B'| L^2}{2M v_x^2}$$

and for $g = 2$,

$$\Delta z = \frac{\mu_B |B'| L^2}{M v_x^2}.$$

This gives the basic quantitative prediction for beam splitting.

What happens after the magnet?

The atoms leave the magnetic field with opposite vertical velocities.

The vertical velocity acquired inside the magnet is

$$v_{z,\pm} = a_{\pm} t.$$

Thus,

$$v_{z,\pm} = \mp \frac{g\mu_B B' L}{2M v_x}.$$

Suppose the detector is a distance D from the exit of the magnet.

The time of flight after leaving the magnet is

$$t_D = \frac{D}{v_x}.$$

The additional displacement is

$$\Delta z_{\text{after}} = v_z t_D.$$

For each beam,

$$z_{\text{after},\pm} = \mp \frac{g\mu_B B' L D}{2M v_x v_x}.$$

Therefore,

$$z_{\text{after},\pm} = \mp \frac{g\mu_B B' L D}{2M v_x^2}.$$

Including the displacement inside the magnet gives

$$z_{\pm} = \mp \frac{g\mu_B B' L^2}{4M v_x^2} \mp \frac{g\mu_B B' L D}{2M v_x^2}.$$

Thus, the total separation at the detector is

$$\Delta z_{\text{detector}} = \frac{g\mu_B |B'|}{2M v_x^2} (L^2 + 2LD)$$

or, for $g = 2$,

$$\Delta z_{\text{detector}} = \frac{\mu_B |B'|}{M v_x^2} (L^2 + 2LD).$$

This is the experimentally useful form for the idealized uniform-gradient model.

Why the spin components separate

The important mathematical structure is

$$\Psi(r, t) = a\psi_+(r, t)\chi_+ + b\psi_-(r, t)\chi_-.$$

Initially,

$$\psi_+(r, 0) = \psi_-(r, 0) = \phi(r).$$

After passing through the magnet,

$$\psi_+(r, t) \neq \psi_-(r, t).$$

Their centers become separated:

$$\langle z \rangle_+ \neq \langle z \rangle_-.$$

The total probability density is

$$P(r, t) = \Psi^\dagger \Psi.$$

Using the orthogonality relation

$$\chi_+^\dagger \chi_- = 0,$$

we get

$$P(r, t) = |a|^2 |\psi_+|^2 + |b|^2 |\psi_-|^2.$$

Thus, the detector records two probability distributions.

Probabilities of the two beams

If the initial spin state is

$$|\chi\rangle = a|+\rangle + b|-\rangle,$$

then the probability of detecting the atom in the +beam is

$$P_+ = |a|^2$$

and the probability of detecting it in the –beam is

$$P_- = |b|^2.$$

Since

$$|a|^2 + |b|^2 = 1,$$

we have

$$P_+ + P_- = 1.$$

General initial spin state: A general spin- $\frac{1}{2}$ state can be written as

$$|\chi\rangle = \cos\frac{\theta}{2}|+\rangle + e^{i\phi}\sin\frac{\theta}{2}|-\rangle.$$

Therefore,

$$a = \cos\frac{\theta}{2}$$

and

$$b = e^{i\phi}\sin\frac{\theta}{2}.$$

Hence,

$$P_+ = \cos^2\frac{\theta}{2}$$

and

$$P_- = \sin^2\frac{\theta}{2}.$$

This is the quantum-mechanical prediction for a Stern–Gerlach measurement along z.

Connection with expectation value

The expectation value of S_z is

$$\langle S_z \rangle = \langle \chi | \hat{S}_z | \chi \rangle.$$

Using

$$\hat{S}_z = \frac{\hbar}{2} \sigma_z,$$

we obtain

$$\langle S_z \rangle = \frac{\hbar}{2} (|a|^2 - |b|^2).$$

For the general state,

$$|a|^2 = \cos^2 \frac{\theta}{2}$$

and

$$|b|^2 = \sin^2 \frac{\theta}{2}.$$

Therefore,

$$\langle S_z \rangle = \frac{\hbar}{2} \left[\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \right].$$

Using

$$\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} = \cos \theta,$$

we get

$$\boxed{\langle S_z \rangle = \frac{\hbar}{2} \cos \theta}.$$

Important role of the Pauli matrices

The Pauli matrices satisfy

$$\boxed{\sigma_i \sigma_j = \delta_{ij} I + i \epsilon_{ijk} \sigma_k}.$$

Therefore,

$$[\sigma_i, \sigma_j] = 2i \epsilon_{ijk} \sigma_k.$$

Consequently,

$$[\hat{S}_i, \hat{S}_j] = i \hbar \epsilon_{ijk} \hat{S}_k.$$

For example,

$$\boxed{[\hat{S}_x, \hat{S}_y] = i \hbar \hat{S}_z}.$$

This means the different components of spin cannot simultaneously possess definite values.

The Stern–Gerlach apparatus selects one component, for example S_z .

Stern–Gerlach apparatus as a quantum measurement: Mathematically, the apparatus implements the projection operators

$$\boxed{P_+ = |+\rangle\langle +|}$$

and

$$\boxed{P_- = |-\rangle\langle -|}.$$

In matrix form,

$$P_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

and

$$P_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

They satisfy

$$P_+ + P_- = I.$$

After the measurement, the state associated with the +detector is

$$|\chi\rangle \rightarrow \frac{P_+ |\chi\rangle}{\sqrt{\langle \chi | P_+ | \chi \rangle}} = |+\rangle$$

and similarly,

$$|\chi\rangle \rightarrow |-\rangle.$$

This is why the experiment is regarded as a physical realization of a quantum measurement of spin.

Non-uniform field is essential. Why? The force is

$$F = \nabla(\mu \cdot B).$$

For the z-direction,

$$F_z = \mu_z \frac{\partial B_z}{\partial z}.$$

If

$$\frac{\partial B_z}{\partial z} = 0,$$

then

$$F_z = 0.$$

The spin can still interact with the field through the energy

$$U = -\mu \cdot B,$$

but there is no spatial splitting.

Therefore,

$$\text{Stern-Gerlach splitting requires a magnetic-field gradient.}$$

Maxwell's equations: A real magnetic field cannot simply be assumed to have only

$$B_z = B_0 + B'z$$

everywhere.

In a source-free region,

$$\nabla \cdot B = 0$$

requires

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0.$$

If

$$\frac{\partial B_z}{\partial z} = B',$$

then the transverse components must have corresponding spatial variations.

For an idealized derivation, however, we focus on the dominant z-force:

$$F_z \simeq \mu_z \frac{\partial B_z}{\partial z}.$$

The central result

For spin- $\frac{1}{2}$,

$$S_z = \pm \frac{\hbar}{2}$$

and therefore, the magnetic moment has two possible projections.

The Stern–Gerlach Hamiltonian is

$$\hat{H}_{\text{SG}} = \frac{\hat{p}^2}{2M} + \frac{g\mu_B}{2} B_z(z) \sigma_z$$

and the force operator is

$$\hat{F}_z = -\frac{g\mu_B}{2} \frac{\partial B_z}{\partial z} \sigma_z.$$

Since

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

there are two eigenvalues,

$$+1, -1.$$

Consequently,

$$F_z = F_0, -F_0$$

Where,

$$F_0 = \frac{g\mu_B}{2} \left| \frac{\partial B_z}{\partial z} \right|.$$

Thus,

$$\text{one beam} \rightarrow \text{two beams}$$

which is the fundamental experimental demonstration of **quantization of spin angular momentum**.

Note: The Stern–Gerlach experiment does not merely show that a magnetic field deflects atoms. Classical electromagnetism already predicts that. Its quantum significance is that the force is correlated with the eigenvalues

$$\sigma_z = \pm 1$$

or equivalently

$$S_z = \pm \frac{\hbar}{2},$$

so, the deflection occurs in discrete amounts rather than continuously.

Conclusion: The experiment can be viewed not merely as a historic experiment but as a pedagogical bridge between classical physics and quantum physics. It helps students move from the idea of a continuously variable classical quantity to the quantum concept of discrete measurement outcomes, while simultaneously introducing spin- $\frac{1}{2}$, operators, eigenvalues, commutation relations and Pauli matrices. These are central elements that need to be known about spin angular momentum. Studying the Stern–Gerlach experiment helps students understand how experimental evidence leads to quantum concepts such as spin, quantization, measurement, and the mathematical operator formalism of quantum mechanics.

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