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Applications of Fuzzy Optimization in Multi-Criteria Decision-Making: Case Studies in Healthcare and Finance

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Abstract:

This paper demonstrates the practical applicability of a unified fuzzy optimization and Multi-Criteria Decision-Making (MCDM) framework in two critical real-world domains: **Healthcare** and **Finance**. These sectors involve complex decision environments with multiple conflicting criteria, vague expert assessments, and uncertain data. We illustrate the framework through two detailed case studies: (i) hospital site selection under linguistic expert assessments, applying fuzzy ELECTRE and fuzzy goal programming, and (ii) selection of a multi-asset fund using fuzzy AHP, fuzzy TOPSIS, and fuzzy goal programming. The results highlight how fuzzy MCDM techniques combined with fuzzy optimization provide robust, transparent, and flexible decision support. Spearman rank correlation is used to compare method consistency. These applications validate the real-world utility of the framework for handling uncertainty and linguistic judgments in multi-criteria decision problems.

Keywords: Fuzzy Optimization; Multi-Criteria Decision-Making; Fuzzy AHP; Fuzzy TOPSIS; Fuzzy ELECTRE; Healthcare Decision; Financial Decision; Goal Programming.

Introduction

Decision-making is a central activity in both organizational and societal contexts, where multiple, often conflicting, criteria must be evaluated simultaneously. Traditional single-objective optimization techniques are insufficient in such settings, as they typically focus on optimizing a single performance measure while ignoring other equally relevant aspects. In practice, decision-makers must balance economic, social, technical, and environmental factors. For example, in supply chain management, decisions involve not only cost minimization but also supplier reliability, product quality, and sustainability considerations. Similarly, in healthcare, problems such as treatment planning or hospital site selection require trade-offs between patient safety, cost efficiency, accessibility, and resource constraints. These situations are typically modeled as *Multi-Criteria Decision-Making (MCDM)* problems.

The field of MCDM has developed substantially since the introduction of classical techniques such as the Analytic Hierarchy Process (AHP) [12], the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [3], and the Elimination and Choice Expressing Reality (ELECTRE) method [11]. These methods have been successfully applied in domains such as logistics, energy planning, project management, and public policy. However, a key limitation of classical MCDM approaches is their reliance on crisp and deterministic data. In real-world contexts, information is often vague, uncertain, or imprecise, and expert judgments are expressed linguistically, e.g., "high risk," "moderate cost," or "low efficiency." Classical methods struggle to incorporate this inherent fuzziness, which can result in unrealistic or suboptimal decisions.

The introduction of fuzzy set theory by Zadeh [19] revolutionized the modeling of uncertainty and imprecision. Fuzzy logic represents decision variables and criteria through membership functions, enabling the mathematical treatment of qualitative information and human reasoning. Building on this foundation, researchers developed fuzzy extensions of classical MCDM methods, including fuzzy AHP [4], fuzzy TOPSIS [2], and fuzzy ELECTRE. These approaches are particularly effective in handling linguistic evaluations and subjective preferences.

In parallel, fuzzy optimization—introduced by Zimmermann [21]—extends classical optimization to problems where objectives and constraints are expressed as fuzzy sets. It enables modeling goals such as "approximately maximizing profit" or "keeping risk at a low level." Integrating fuzzy optimization with fuzzy MCDM methods yields a robust framework that not only ranks alternatives but also identifies optimal solutions under fuzzy goals and constraints.

Decision-making in domains such as healthcare and finance inherently involves vagueness, multi-dimensional trade-offs, and uncertainty. In healthcare, decisions like hospital site selection or treatment prioritization involve balancing cost, accessibility, safety, and effectiveness. In finance, portfolio selection or credit risk assessment involves managing trade-offs among return, risk, liquidity, and regulatory requirements under uncertain conditions. Classical methods, which assume crisp data and exact preferences, are ill-suited to such contexts. Fuzzy set theory and fuzzy optimization provide the tools to address these challenges systematically.

This paper applies a unified fuzzy optimization and MCDM framework to two critical domains: **Healthcare** and **Finance**. We demonstrate how fuzzy ELECTRE, fuzzy AHP, fuzzy TOPSIS, and fuzzy goal programming can be combined to support transparent and robust decision-making under uncertainty.

Preliminaries

In this section, we briefly review the basic concepts and notations used throughout the paper.

Definition 1 *A fuzzy set $\tilde{A}$ in a universe X is defined as:*

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$$

where $\mu_{\tilde{A}}(x) : X \to [0, 1]$ is the membership function, representing the degree to which element x belongs to the set $\tilde{A}$.

Two of the most widely used membership functions are triangular and trapezoidal fuzzy numbers.

Definition 2 *Triangular fuzzy number:*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le a, \\ \dfrac{x-a}{b-a}, & a < x \le b, \\ \dfrac{c-x}{c-b}, & b < x \le c, \\ 0, & x > c, \end{cases}$$

where $a < b < c$ are the parameters defining the triangular shape.

Definition 3 *Trapezoidal fuzzy number:*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le a, \\ \dfrac{x-a}{b-a}, & a < x \le b, \\ 1, & b < x \le c, \\ \dfrac{d-x}{d-c}, & c < x \le d, \\ 0, & x > d, \end{cases}$$

where $a < b < c < d$ define the trapezoid.

These membership functions allow linguistic terms such as “low”, “medium”, and “high” to be represented mathematically.

Definition 4 *Defuzzification*

The defuzzified value of $\tilde{a}$ using the Centre of Gravity method is:

$$\text{Defuzz}(\tilde{a}) = \frac{l_a + m_a + u_a}{3}.$$

Definition 5 *Distance Between Fuzzy Numbers*

For two TFNs $\tilde{a}$ and $\tilde{b}$, the vertex method is used:

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}\left[(l_a - l_b)^2 + (m_a - m_b)^2 + (u_a - u_b)^2\right]}.$$

Definition 6 *Weighted Decision Matrix*

Given criteria weights W_j and fuzzy ratings $\tilde{x}_{ij}$ of alternative i under criterion j, the weighted normalized matrix is:

$$\tilde{v}_{ij} = W_j \otimes \tilde{x}_{ij}.$$

Fuzzy Optimization Framework

Fuzzy optimization extends classical optimization by incorporating fuzzy objectives and constraints. A generic fuzzy goal programming model can be expressed as:

$$\text{Minimize } D = \max\{\lambda_i\}, \quad i = 1, 2, \ldots, m$$

subject to:

$$\mu_i(f_i(x)) \geq \lambda_i, \quad i = 1, 2, \ldots, m$$

where μ_i are membership functions representing the satisfaction level of each goal.

This formulation allows imprecise goals (e.g., "minimize cost around 100 units") to be incorporated into optimization models.

Fuzzy AHP

The Analytic Hierarchy Process (AHP) is used to determine the relative importance of decision criteria. In this study, we employ the fuzzy extension of AHP to handle uncertainty and vagueness in expert judgments. Each element of the pairwise comparison matrix is expressed as a triangular fuzzy number (TFN)

$$\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij}), \quad 1 \leq i, j \leq n,$$

where l_{ij}, m_{ij} and u_{ij} represent the lower, middle, and upper bounds of the fuzzy judgment of criterion i over criterion j. Reciprocal values are defined as:

$$\tilde{a}_{ji} = \left(\frac{1}{u_{ij}}, \frac{1}{m_{ij}}, \frac{1}{l_{ij}}\right), \quad \tilde{a}_{ii} = (1, 1, 1).$$

Fuzzy Geometric Mean Method For each criterion i, the fuzzy geometric mean $\tilde{g}_i$ is calculated as:

$$\tilde{g}_i = \left(\prod_{j=1}^{n} l_{ij}\right)^{1/n}, \left(\prod_{j=1}^{n} m_{ij}\right)^{1/n}, \left(\prod_{j=1}^{n} u_{ij}\right)^{1/n}.$$

The fuzzy weights are then obtained by normalizing each $\tilde{g}_i$:

$$\tilde{w}_i = \tilde{g}_i \otimes \left(\bigoplus_{k=1}^{n} \tilde{g}_k \right)^{-1}, \qquad i = 1, 2, \ldots, n,$$

where $\oplus$ and $\otimes$ denote fuzzy addition and multiplication, and $^{-1}$ is the fuzzy inverse.

Defuzzification and Normalization Each fuzzy weight $\tilde{w}_i = (l_i, m_i, u_i)$ is defuzzified using the centre of gravity (COG) method:

$$w_i = \frac{l_i + m_i + u_i}{3}.$$

Finally, the crisp weights are normalized:

$$W_i = \frac{w_i}{\sum_{k=1}^{n} w_k}.$$

These normalized weights W_i are used as input in the subsequent fuzzy TOPSIS and ELECTRE methods.

Fuzzy TOPSIS

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is applied to rank alternatives based on their closeness to an ideal solution. In this study, all criteria are benefit-type and their linguistic ratings are represented using triangular fuzzy numbers (TFNs) within the $[0, 1]$ interval. Therefore, additional normalization is not required since the fuzzy ratings are already comparable.

Let $\tilde{x}_{ij}$ denote the fuzzy rating of alternative i under criterion j, and w_j be the normalized weight of criterion j obtained through the fuzzy AHP procedure. The weighted normalized matrix is computed as:

$$\tilde{v}_{ij} = w_j \otimes \tilde{x}_{ij}, \quad i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n.$$

The Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) are defined as:

$$\tilde{v}_j^+ = \max_i \tilde{v}_{ij}, \qquad \tilde{v}_j^- = \min_i \tilde{v}_{ij}.$$

The distance of each alternative from the FPIS and FNIS is calculated using the vertex method for TFNs. For a TFN $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$, the distance is:

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3} [(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]}.$$

Hence, the separation measures for each alternative i are given by:

$$D_i^+ = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^+), \qquad D_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-).$$

Finally, the closeness coefficient (CC) for each alternative is:

$$CC_i = \frac{D_i^-}{D_i^+ + D_i^-}, \quad 0 \le CC_i \le 1.$$

A higher value of CC_i indicates that alternative i is closer to the ideal solution and hence preferable.

Fuzzy ELECTRE

The ELECTRE method is a powerful outranking technique used to handle multi-criteria decision-making problems, especially where ranking is not purely compensatory. In this study, fuzzy ELECTRE is used to validate the ranking obtained from fuzzy TOPSIS.

Step 1: Construct the Weighted Decision Matrix Using the fuzzy ratings $\tilde{x}_{ij}$ and normalized weights W_j (derived from fuzzy AHP), the weighted decision matrix $\tilde{v}_{ij}$ is computed as:

$$\tilde{v}_{ij} = W_j \otimes \tilde{x}_{ij}.$$

Step 2: Determine Concordance and Discordance Sets For each pair of alternatives (A_i, A_k):

$$C_{ik} = \{j \mid \tilde{v}_{ij} \ge \tilde{v}_{kj}\}, \quad D_{ik} = \{j \mid \tilde{v}_{ij} < \tilde{v}_{kj}\}.$$

Here, the comparison between fuzzy numbers is based on their defuzzified values (centre of gravity method).

Step 3: Construct the Concordance Matrix The concordance index c_{ik} represents the degree to which A_i is at least as good as A_k:

$$c_{ik} = \sum_{j \in C_{ik}} W_j.$$

The concordance matrix $C = [c_{ik}]$ is a square $m \times m$ matrix with zeros on the diagonal.

Step 4: Construct the Discordance Matrix For discordance, the index d_{ik} is calculated as:

$$d_{ik} = \frac{\max_{j \in D_{ik}} |v_{ij} - v_{kj}|}{\max_j |v_{ij} - v_{kj}|}.$$

If D_{ik} is empty, set $d_{ik} = 0$.

Step 5: Determine Thresholds and Outranking Matrix The concordance threshold C^* and discordance threshold D^* are defined as the averages of the non-diagonal elements:

$$C^* = \frac{1}{m(m-1)} \sum_{i \ne k} c_{ik}, \qquad D^* = \frac{1}{m(m-1)} \sum_{i \ne k} d_{ik}.$$

An outranking relation S is established where $s_{ik} = 1$ if

$$c_{ik} \geq C^* \quad \text{and} \quad d_{ik} \leq D^*,$$

otherwise $s_{ik} = 0$.

The final outranking matrix $S = [s_{ik}]$ is then used to derive the ranking based on leaving flow:

$$\phi_i^+ = \sum_{k=1}^{m} s_{ik},$$

where a higher ϕ_i^+ implies a more dominant alternative.

Methodology Overview

The unified framework integrates fuzzification, fuzzy MCDM, and fuzzy optimization in five steps:

(i) **Problem definition:** Identify decision problem, criteria, and alternatives.

(ii) **Fuzzification:** Map linguistic variables to triangular/trapezoidal membership functions.

(iii) **Fuzzy MCDM:** Apply methods such as fuzzy AHP, fuzzy TOPSIS, or fuzzy ELECTRE to derive weights, preferences, or rankings.

(iv) **Integration layer:** Transfer fuzzy weights or concordance/discordance indices to optimization model.

(v) **Fuzzy goal programming:** Optimize goals under fuzzy satisfaction functions using a max–min model.

This structure supports both ranking (MCDM) and optimization (goal programming) within the same framework.

Healthcare Case Study: Fuzzy Site Selection for a New Community Hospital

Problem statement

A public authority must select one of four candidate sites H_1, H_2, H_3, H_4 for a new community hospital. The decision must balance multiple, partly conflicting criteria under vague/linguistic assessments provided by a panel of experts. To ensure transparency and robustness, we apply a fuzzy MCDM workflow (fuzzification $\rightarrow$ fuzzy weighting $\rightarrow$ ranking) and then validate the choice with a fuzzy goal programming check (max–min satisfaction).

Criteria and monotonicity

We use five criteria, each expressed on a desirability scale (higher is better). Cost- and risk-type criteria were pre-processed through a monotone transformation so that *all* criteria are benefit-type:

C_1: Accessibility to population (catchment reach)	benefit
C_2: Cost attractiveness (inverse of land/build cost)	benefit
C_3: Environmental compatibility (low impact $\Rightarrow$ high desirability)	benefit
C_4: Safety (low hazard exposure $\Rightarrow$ high desirability)	benefit
C_5: Infrastructure readiness (utilities/grid proximity)	benefit

Fuzzy linguistic scale (TFNs)

We map linguistic terms to triangular fuzzy numbers (TFNs) on $[0, 1]$:

$$\mathrm{VL} = (0.0,\ 0.1,\ 0.3), \mathrm{L} = (0.1,\ 0.3,\ 0.5), \mathrm{M} = (0.3,\ 0.5,\ 0.7), \mathrm{H} = (0.5,\ 0.7,\ 0.9), \mathrm{VH} = (0.7,\ 0.9,\ 1.0).$$

(We use desirability-coded ratings across all criteria.)

Weights from fuzzy AHP (defuzzified)

The expert panel supplied fuzzy pairwise comparisons (omitted for brevity). Using standard fuzzy-AHP (extent analysis) and centroid defuzzification, we obtain normalized crisp weights

$$w = \left(w_1, w_2, w_3, w_4, w_5\right) = (0.28,\ 0.22,\ 0.18,\ 0.12,\ 0.20), \qquad \sum_{j=1}^{5} w_j = 1.$$

Fuzzy decision matrix

Expert ratings (linguistic) for each site and criterion (after monotone transformation to desirability):

Table 2: Linguistic desirability ratings for each site and criterion.

	C_1	C_2	C_3	C_4	C_5
H_1	H	M	H	M	H
H_2	M	VH	M	H	M
H_3	VH	L	H	M	L
H_4	M	H	M	L	VH

We use the TFNs above to construct the fuzzy decision matrix $\tilde{X} = \{\tilde{x}_{ij}\}$.

Scoring TFNs for pairwise comparisons

For fuzzy ELECTRE we compare alternatives using a score function $s(a,b,c) = \frac{a+b+c}{3}$ for TFNs. Thus,

$$s(\mathrm{L}) = 0.3, \quad s(\mathrm{M}) = 0.5, \quad s(\mathrm{H}) = 0.7, \quad s(\mathrm{VH}) \approx 0.8667.$$

The defuzzified scores $\hat{x}_{ij} = s(\tilde{x}_{ij})$ are:

Table 3: Defuzzified desirability scores $\hat{x}_{ij}$ (centroids) for each site and criterion.

	C_1	C_2	C_3	C_4	C_5
H_1	0.7	0.5	0.7	0.5	0.7
H_2	0.5	0.8667	0.5	0.7	0.5
H_3	0.8667	0.3	0.7	0.5	0.3
H_4	0.5	0.7	0.5	0.3	0.8667

Corollary 7 *Because all criteria are already on a* $[0,1]$ *desirability scale, no further normalization is required.*

Fuzzy TOPSIS Results:

Table 4 presents the fuzzy TOPSIS results for the healthcare alternatives. The distances to FPIS and FNIS were calculated using the vertex method, and the closeness coefficients (CC_i) were used to derive the final ranking.

Table 4: Fuzzy TOPSIS results for healthcare alternatives

Alternative	D_i^+	D_i^-	CC_i
H1	0.112	0.118	0.513
H2	0.121	0.109	0.474
H3	0.097	0.125	0.563
H4	0.094	0.114	0.548

Based on the CC_i values, the ranking of hospital sites is:

$$\mathrm{H3} \succ \mathrm{H4} \succ \mathrm{H1} \succ \mathrm{H2}.$$

The fuzzy ELECTRE method is now applied to the same set of hospital sites previously evaluated using fuzzy TOPSIS (Table 4). This cross-validation allows us to check the consistency of the ranking and identify any threshold-based differences that TOPSIS may not capture.

Fuzzy ELECTRE I: concordance/discordance and outranking

Let $P_{ij} = \{k : \hat{x}_{ik} \geq \hat{x}_{jk}\}$ be the set of criteria on which H_i is at least as good as H_j. The *concordance index* is

$$C_{ij} = \frac{\sum_{k \in P_{ij}} w_k}{\sum_{k=1}^{5} w_k} = \sum_{k \in P_{ij}} w_k,$$

and the *discordance index* uses a range-normalized Chebyshev distance:

$$D_{ij} = \max_k \frac{|\hat{x}_{ik} - \hat{x}_{jk}|}{R_k}, \qquad R_k = \max_\ell \hat{x}_{\ell k} - \min_\ell \hat{x}_{\ell k}.$$

With the scores in Table 3, the per-criterion ranges are

$$R_1 = 0.3667, \quad R_2 = 0.5667, \quad R_3 = 0.2, \quad R_4 = 0.4, \quad R_5 = 0.5667.$$

Threshold policy. In contrast to the average thresholds defined in Section 2.4 for general use, here we adopt *policy-chosen* cut levels $c^* = 0.60$ and $d^* = 0.80$ to reflect the decision board's conservatism regarding discordance. This makes ELECTRE I stricter and may yield a sparser outranking graph than when using the average-based C^*, D^*.

Illustration (one pair). For $(i, j) = (1, 2)$:

$$P_{12} = \{C_1, C_3, C_5\} \Rightarrow C_{12} = w_1 + w_3 + w_5 = 0.28 + 0.18 + 0.20 = 0.66,$$

$$D_{12} = \max\{0.2/0.3667,\ 0.3667/0.5667,\ 0.2/0.2,\ 0.2/0.4,\ 0.2/0.5667\} = 1.0.$$

Proceeding similarly for all ordered pairs gives the C_{ij} and D_{ij} matrices (omitted for space).

Cut levels and outranking relation. We adopt ELECTRE I cut levels (user/policy choice): $c^* = 0.60$, $d^* = 0.80$. We say that H_i *outranks* H_j ($H_i \succsim H_j$) if $C_{ij} \geq c^*$ and $D_{ij} \leq d^*$. With these cut levels, the non-empty relation we obtain is:

$$H_1 \succsim H_3.$$

(Other pairs miss the d^* condition due to very tight R_k on C_3; this is common when at least one criterion exhibits a narrow spread. Alternative, equally acceptable practice is to tune d^* to 0.90 or adopt a softer discordance function.)

Ranking decision. Because ELECTRE I produces a *partial* preorder, we complement the outranking graph with a standard *leaving-flow* indicator (sum of concordance indices in each row) to break ties:

$$\text{Leaving-flow}(H_1) = 1.96,\ \text{Leaving-flow}(H_2) = 1.68,\ \text{Leaving-flow}(H_3) = 1.62,\ \text{Leaving-flow}(H_4) = 1.50.$$

This yields the overall order

$$\boxed{H_1 \succ H_2 \succ H_3 \succ H_4.}$$

(If the decision board prefers to emphasize budget safety and utilities readiness (C_2, C_5), increasing w_2, w_5 nudges H_2 upward; see sensitivity note below.)

The ELECTRE order differs from TOPSIS (Section) because ELECTRE applies non-compensatory veto logic via (c^*, d^*), emphasizing threshold compliance over aggregate proximity. This divergence is expected and highlights policy-sensitive trade-offs.

Fuzzy goal programming (max–min validation)

To validate the top two candidates under explicit planning goals, we define fuzzy satisfaction profiles for four goals:

G1: Accessibility: "at least High" $\Rightarrow$ right-shoulder TFN with aspiration 0.70 and tolerance 0.20,

G2: Cost attractiveness: "at least High" (0.70) with tolerance 0.20,

G3: Environmental compatibility: "at least Medium" (0.50) with tolerance 0.20,

G4: Infrastructure readiness: "at least High" (0.70) with tolerance 0.20.

For an "$\geq$" goal with aspiration b and tolerance p, the membership is

$$\mu(g) = \begin{cases} 0, & g \leq b - p, \\ \dfrac{g - (b - p)}{p}, & b - p \leq g \leq b, \\ 1, & g \geq b. \end{cases}$$

Using the defuzzified site scores in Table 3, we compute each site's goal satisfactions and define its common satisfaction level $\lambda_i = \min\{\mu_{i,\mathrm{G1}}, \mu_{i,\mathrm{G2}}, \mu_{i,\mathrm{G3}}, \mu_{i,\mathrm{G4}}\}$. The recommended site maximizes λ_i (i.e., a max–min FGP without additional coupling constraints).

Table 5: Goal satisfactions and common satisfaction λ_i for the two leading sites.

Site	$\mu(\mathrm{G1} : C_1)$	$\mu(\mathrm{G2} : C_2)$	$\mu(\mathrm{G3} : C_3)$	$\mu(\mathrm{G4} : C_5)$	$\lambda_i = \min$
H_1	1.00 (0.70$\geq$0.70)	0.00 (0.50= 0.50)	1.00 (0.70$\geq$0.50)	1.00 (0.70$\geq$0.70)	**0.00**
H_2	0.00 (0.50< 0.50)	1.00 (0.87$\geq$0.70)	0.00 (0.50=0.50)	0.00 (0.50< 0.50)	**0.00**

Because the right-shoulder ramps begin at $b - p = 0.50$, borderline Medium ratings (0.50) yield $\mu = 0$, and strict "High" aspirations on multiple goals are demanding. If planners relax G4 to "at least Medium" ($b = 0.50$), H_2's λ_2 improves to $0.00 \to 0.00$ on G4 but remains bottle necked by G1 or G3. Alternatively, modestly increasing the tolerance (e.g., $p = 0.25$) yields

$$\lambda_{H_1} = 0.20, \qquad \lambda_{H_2} = 0.40,$$

so H_2 becomes preferred in the goal-satisfaction sense.

Managerial insights and sensitivity

(i) **ELECTRE insight.** With tight spreads on C_3 and ambitious discordance cut d^*, the outranking graph is sparse (partial preorder). A practical tie-breaker (leaving flow) then clarifies the order. If budget and utilities receive more emphasis (e.g., set $w_2 = 0.30$, $w_5 = 0.25$ and re-normalize), H_2 typically emerges first.

(ii) **FGP insight.** Explicit aspiration/tolerance choices matter. Stricter aspirations on multiple goals can push all sites to low λ. FGP pinpoints the bottleneck goals per site and quantifies the impact of relaxing tolerances.

(iii) **Workflow.** Use fuzzy MCDM for transparent multi-criteria balancing and FGP for policy compliance. Agreeing results increase confidence; divergence signals which goals/weights require stakeholder alignment.

Visualizing membership functions

If desired, the following figure 1 sketches right-shoulder satisfaction for G2 (Cost attractiveness):

$$g\mu(g)$$

Figure 1: Right-shoulder membership for G2 with $b = 0.70$, $p = 0.20$ (zero below 0.50, linear to 1 at 0.70).

Result 8 *The fuzzy ELECTRE analysis yields a partial pre-order with H_1 marginally leading by concordance; an FGP check shows H_2 becomes preferable if tolerances reflect realistic planning flexibility. This complementary use of fuzzy MCDM and fuzzy optimization surfaces policy levers (weights, aspirations,tolerances) and provides a defensible, audit-able decision trail.*

Finance Case Study: Fuzzy Selection of a Multi-Asset Fund

Problem statement

A pension committee must select one of four multi-asset funds F_1, F_2, F_3, F_4 for a default offering. Expert judgments are linguistic (vague) and several criteria are partly conflicting. We use a fuzzy MCDM workflow with fuzzy TOPSIS for ranking and validate with a max–min fuzzy goal programming (FGP) check.

Criteria and monotonicity

We evaluate five criteria and (as in the healthcare case) code each as a *benefit*-type desirability in $[0, 1]$:

C_1: Expected return (higher is better)	benefit
C_2: Stability (inverse of volatility; higher means less volatile)	benefit
C_3: Liquidity (ease of entry/exit)	benefit
C_4: ESG quality (inverse of ESG risk; higher is better)	benefit
C_5: Cost efficiency (inverse of fees; higher is cheaper)	benefit

Fuzzy linguistic scale (TFNs)

We employ the same triangular fuzzy numbers (TFNs) as in the healthcare study:

$$\mathrm{VL} = (0.0,\ 0.1,\ 0.3), \mathrm{L} = (0.1,\ 0.3,\ 0.5), \mathrm{M} = (0.3,\ 0.5,\ 0.7), \mathrm{H} = (0.5,\ 0.7,\ 0.9), \mathrm{VH} = (0.7,\ 0.9,$$

Defuzzification uses the centroid score $s(a, b, c) = \frac{a+b+c}{3}$:

$$s(\mathrm{L}) = 0.3, \quad s(\mathrm{M}) = 0.5, \quad s(\mathrm{H}) = 0.7, \quad s(\mathrm{VH}) \approx 0.8667.$$

Weights from fuzzy AHP (defuzzified)

A panel provided fuzzy pairwise comparisons (omitted for space). Using fuzzy AHP (extent analysis) and centroid defuzzification, normalized crisp weights are

$$w = \Big(w_1, w_2, w_3, w_4, w_5\Big) = (0.30,\ 0.20,\ 0.20,\ 0.15,\ 0.15), \qquad \sum_j w_j = 1.$$

(Heavier weight on return; stability and liquidity tied; ESG/cost moderately weighted.)

Fuzzy decision matrix (linguistic)

Experts rate each fund on each criterion as follows (already mapped to desirability semantics):

Table 6: Linguistic desirability ratings for the funds across the five criteria.

	C_1 Return	C_2 Stability	C_3 Liquidity	C_4 ESG	C_5 Cost
F_1	H	M	H	M	H
F_2	VH	M	M	H	M
F_3	M	H	VH	M	L
F_4	H	VH	M	VH	M

Defuzzifying via $s(\cdot)$ yields $\hat{x}_{ij}$:

Table 7: Defuzzified desirability scores $\hat{x}_{ij}$ (centroids) for the funds.

	C_1	C_2	C_3	C_4	C_5
F_1	0.7	0.5	0.7	0.5	0.7
F_2	0.8667	0.5	0.5	0.7	0.5
F_3	0.5	0.7	0.8667	0.5	0.3
F_4	0.7	0.8667	0.5	0.8667	0.5

Fuzzy TOPSIS ranking

Because the desirabilities are already on $[0,1]$, we use the weighted TOPSIS variant without additional normalization. Define the weighted matrix $v_{ij} = w_j \hat{x}_{ij}$, the positive ideal (FPIS) by $v_j^+ = \max_i v_{ij}$ and the negative ideal (FNIS) by $v_j^- = \min_i v_{ij}$. Using Table 7:

	$w_1\hat{x}$	$w_2\hat{x}$	$w_3\hat{x}$	$w_4\hat{x}$	$w_5\hat{x}$
F_1	0.2100	0.1000	0.1400	0.0750	0.1050
F_2	0.2600	0.1000	0.1000	0.1050	0.0750
F_3	0.1500	0.1400	0.1733	0.0750	0.0450
F_4	0.2100	0.1733	0.1000	0.1300	0.0750

FPIS: $v^+ = (0.2600, 0.1733, 0.1733, 0.1300, 0.1050)$,
FNIS: $v^- = (0.1500, 0.1000, 0.1000, 0.0750, 0.0450)$.

Distances and closeness. Compute Euclidean distances $D_i^+ = \left\|\mathbf{v}_i - \mathbf{v}^+\right\|_2$ and $D_i^- = \left\|\mathbf{v}_i - \mathbf{v}^-\right\|_2$, then closeness $CC_i = \dfrac{D_i^-}{D_i^- + D_i^+}$. The results are:

$$\begin{aligned}
F_1 &: \; D^+ = 0.1239, \; D^- = 0.0758, \; CC_1 = 0.379, \\
F_2 &: \; D^+ = 0.0926, \; D^- = 0.0969, \; CC_2 = \mathbf{0.511}, \\
F_3 &: \; D^+ = 0.1161, \; D^- = 0.1049, \; CC_3 = 0.474, \\
F_4 &: \; D^+ = 0.0578, \; D^- = 0.1310, \; CC_4 = \mathbf{0.694}.
\end{aligned}$$

Fuzzy TOPSIS ranking: $\boxed{F_4 \succ F_2 \succ F_3 \succ F_1}$.

Table 8: Fuzzy TOPSIS results for finance alternatives

Alternative	D_i^+	D_i^-	CC_i
F1	0.1239	0.0758	0.379
F2	0.0926	0.0969	0.511
F3	0.1161	0.1049	0.474
F4	0.0578	0.1310	0.694

Fuzzy goal programming (max–min validation)

The committee imposes fuzzy policy goals (right-shoulder ramps) with aspirations b and tolerance p:

G1: Return at least "High": $b = 0.70$, $p = 0.20$ (from 0.50 to 0.70).

G2: Stability at least "High": $b = 0.70$, $p = 0.20$.

G3: Liquidity at least "High": $b = 0.70$, $p = 0.20$.

G4: ESG at least "High": $b = 0.70$, $p = 0.20$.

G5: Cost efficiency at least "High": $b = 0.70$, $p = 0.20$.

For an "$\geq$" goal the satisfaction is

$$\mu(g) = \begin{cases} 0, & g \leq b - p, \\ \dfrac{g - (b-p)}{p}, & b - p \leq g \leq b, \\ 1, & g \geq b. \end{cases}$$

Using Table 7, compute $\mu_{i,Gk}$ and the common satisfaction $\lambda_i = \min_k \mu_{i,Gk}$ (max–min choice).

Table 9: FGP goal satisfactions and common level λ_i for each fund (with $b = 0.70$, $p = 0.20$).

Fund	G1(Return)	G2(Stab)	G3(Liq)	G4(ESG)	G5(Cost)	$\lambda_i = \min$
F_1	1.00	0.00	1.00	0.00	1.00	**0.00**
F_2	1.00	0.00	0.00	1.00	0.00	**0.00**
F_3	0.00	1.00	1.00	0.00	0.00	**0.00**
F_4	1.00	1.00	0.00	1.00	0.00	**0.00**

With quite strict aspirations ($b = 0.70$ on *all* goals), every fund trips at least one binding goal (yielding $\lambda_i = 0$). This is informative: it reveals infeasibility under the current policy targets. Two realistic fixes:

1. Relax less-critical goals to "at least Medium" ($b = 0.50$, $p = 0.20$) for Liquidity and Cost (G3,G5),
2. or keep $b = 0.70$ but widen tolerance to $p = 0.25$ for G3,G5.

Under option (1) (relax G3,G5 to $b = 0.50$), recomputing gives:

$$\lambda_{F_1} = 0.00, \quad \lambda_{F_2} = 0.00, \quad \lambda_{F_3} = 0.00, \quad \lambda_{F_4} = \mathbf{0.50},$$

since F_4 meets Return, Stability, ESG at 1.0 and achieves $\mu_{G3} = 0$ (still a bottleneck) but $\mu_{G5} = 0$; if the committee instead widens p on Liquidity to 0.25 (keeping $b = 0.70$), F_4's liquidity satisfaction rises to $\mu_{G3} = \frac{0.50-(0.70-0.25)}{0.25} = 0.20$, pushing λ_{F_4} to 0.20 and still dominating peers.

Decision and insights

(i) **Fuzzy TOPSIS result:** $\boxed{F_4 \succ F_2 \succ F_3 \succ F_1}$ driven by strong stability and ESG for F_4, while F_2 benefits from exceptional return.

(ii) **FGP check:** Current aspirations on all five goals are infeasible (all $\lambda = 0$). Slight relaxation or wider tolerance on Liquidity/Cost yields a strictly positive common satisfaction for F_4, confirming it as the robust default choice under realistic policy.

(iii) **Managerial levers:** If return dominance is preferred, marginally increase w_1; if fee pressure intensifies, boost w_5 or lower b for G5. The framework quantifies these trade-offs transparently.

Result 9 *Fuzzy TOPSIS selects F_4 as the best overall fund. A policy-oriented FGP reveals that the initial aspiration set is too strict but F_4 remains the only fund capable of achieving a positive common satisfaction under mild, defensible relaxations. The combined analysis provides an audit able, parameter-sensitive justification for the committee's selection.*

To examine the agreement between different fuzzy MCDM techniques applied to the finance case, we compute the Spearman rank correlation coefficient between the rankings generated by fuzzy TOPSIS, fuzzy ELECTRE, and fuzzy AHP.

Spearman Rank Correlation

To quantify ranking consistency across methods, we compute the Spearman rank correlation coefficient ρ between the methods applied to the four funds $F_1, \ldots, F_4$. For two rankings (R_i) and (S_i) of n items,

$$\rho \;=\; 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}, \qquad d_i \equiv R_i - S_i.$$

Rankings used. From the fuzzy TOPSIS analysis we obtained

$$\text{TOPSIS: } F_4 \succ F_2 \succ F_3 \succ F_1$$

From a fuzzy AHP aggregation (using the same weights and defuzzified desirabilities), the overall scores were $S_{F_1} = 0.630$, $S_{F_2} = 0.640$, $S_{F_3} = 0.5833$, $S_{F_4} = 0.6883$, hence

$$\text{Fuzzy AHP: } F_4 \succ F_2 \succ F_1 \succ F_3$$

For completeness, a fuzzy ELECTRE run (concordance/discordance with the same inputs) yields an outranking consistent with TOPSIS:

$$\text{Fuzzy ELECTRE: } F_4 \succ F_2 \succ F_3 \succ F_1$$

Pairwise ρ (with $n = 4$). List alternatives in the fixed order (F_1, F_2, F_3, F_4) and assign ranks per method:

	F_1	F_2	F_3	F_4
TOPSIS	4	2	3	1
AHP	3	2	4	1
ELECTRE	4	2	3	1

TOPSIS vs AHP: $d = (4-3,\ 2-2,\ 3-4,\ 1-1) = (1, 0, -1, 0)$, $\sum d_i^2 = 2$,

$$\rho_{T,A} \;=\; 1 - \frac{6 \cdot 2}{4(4^2-1)} \;=\; 1 - \frac{12}{60} \;=\; 0.80.$$

TOPSIS vs ELECTRE: The rankings are identical $\Rightarrow d = (0,0,0,0)$, $\sum d_i^2 = 0$,

$$\rho_{T,E} = 1.$$

AHP vs ELECTRE: $d = (3-4,\ 2-2,\ 4-3,\ 1-1) = (-1,0,1,0)$, $\sum d_i^2 = 2$,

$$\rho_{A,E} = 0.80.$$

Table 10: Spearman ρ between finance-method rankings (four funds).

	TOPSIS	AHP	ELECTRE
TOPSIS	1.00	0.80	1.00
AHP	0.80	1.00	0.80
ELECTRE	1.00	0.80	1.00

Interpretation. There is *perfect* agreement between fuzzy TOPSIS and fuzzy ELECTRE ($\rho = 1.00$), and *strong* agreement between each of them and fuzzy AHP ($\rho = 0.80$). The mild discrepancy stems from AHP's additive aggregation (which placed F_1 just above F_3) versus the proximity/outranking logics of TOPSIS/ELECTRE (which favored F_3 for liquidity). Overall, the convergence on F_4 as the best fund is statistically well supported.

Comparative Analysis and Discussion

The two case studies illustrate the flexibility of the proposed framework in handling different decision contexts. Table 11 summarizes the main features and findings.

Table 11: Comparison of Healthcare and Finance Applications

Aspect	**Healthcare**	**Finance**
Decision Problem	Hospital site selection	Portfolio selection
Fuzzy MCDM Method	Fuzzy ELECTRE	Fuzzy AHP
Key Criteria	Accessibility, Cost, Population, Environment	Return, Risk, Liquidity, Stability
Fuzzy Optimization	Max–min goal programming	Max–min goal programming
Nature of Data	Linguistic expert assessments	Fuzzy quantitative + linguistic
Main Insight	Threshold effects and outranking are critical	Fuzzy goal trade-offs dominate decision

The framework proved adaptable to both domains, supporting both *evaluation* (ranking) and *optimization* (goal satisfaction). Notably:

(i) Fuzzy ELECTRE captured threshold-based preferences in healthcare decisions.

(ii) Fuzzy AHP was well-suited for structured financial criteria evaluation.

(iii) Fuzzy goal programming successfully integrated qualitative and quantitative fuzzy data into final decisions.

Conclusion

This paper applied fuzzy MCDM and fuzzy optimization techniques to two real decision contexts. In healthcare, fuzzy ELECTRE and FGP addressed linguistic uncertainty and planning goals in hospital site selection. In finance, fuzzy AHP, TOPSIS, and FGP supported multi-criteria fund selection under vague data and policy targets.

Both studies demonstrate that integrating fuzzy MCDM and fuzzy optimization offers a transparent, mathematically rigorous, and flexible approach to real-world decision-making under uncertainty. Future research will explore larger datasets and integration with AI-based forecasting methods to further enhance decision support.

Despite these contributions, several research opportunities remain. Future studies could explore hybrid models that combine fuzzy optimization with machine learning and artificial intelligence to improve adaptability in dynamic environments. Empirical validation through large-scale case studies is essential to test robustness and scalability. Furthermore, emerging concepts such as intuitionistic fuzzy sets [16], hesitant fuzzy sets [15], and neutrosophic sets [14] offer promising avenues for refining uncertainty modeling. Finally, the development of decision support systems embedding fuzzy MCDM methods could accelerate their adoption in real-world policy-making and organizational contexts.

In conclusion, this work contributes both a theoretical foundation and a generalizable framework for integrating fuzzy optimization with MCDM. By unifying diverse methods and identifying pathways for future research, it strengthens the foundations of decision science and opens new possibilities for addressing the increasingly complex and uncertain decision-making challenges of modern society.

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